A permutation $\pi$ on a set of positive integers $\{a_1, a_2, ..., a_n\}$ is said to be graphical if there exists a graph containing exactly $a_i$ vertices of degree $\pi(a_i)$. It has been shown that for positive integers with $a_1 < a_2 < ... < a_n$, if $\pi(a_n) = a_n$, then the permutation $\pi$ is graphical if and only if the sum $\sum_{i=1}^{n} a_i \pi(a_i)$ is even and $a_n \leq \sum_{i=1}^{n-1} a_i \pi(a_i)$. This known result has been proved using a criterion of Fulkerson, Hoffman, and McAndrew which requires the verification of $\frac{1}{2}n(n+1)$ inequalities. In this talk, we use a criterion of Tripathi and Vijay to give a shorter proof of this result, requiring only the verification of $n$ inequalities. We also use this criterion to provide a similar result for permutations $\pi$ such that $\pi(a_{n-1}) = a_n$. We prove that such a permutation is graphical if and only if the sum $\sum_{i=1}^{n} a_i \pi(a_i)$ is even and $a_n a_{n-1} \leq a_{n-1}(a_{n-1} - 1) + \sum_{i \neq n-1} a_i \pi(a_i)$. We also consider permutations such that $\pi(a_n) = a_{n-1}$, and then more generally, permutations such that $\pi(a_n) = a_{n-j}$ for some $j < n$. A Study of Graphical Permutations

Jessica Thune*

Department of Mathematical Sciences, University of Nevada Las Vegas
Las Vegas, NV 89154
thunej@unlv.nevada.edu

A Study of Graphical Permutations

Jessica Thune*

Department of Mathematical Sciences, University of Nevada Las Vegas
Las Vegas, NV 89154
thunej@unlv.nevada.edu

A permutation $\pi$ on a set of positive integers $\{a_1, a_2, ..., a_n\}$ is said to be graphical if there exists a graph containing exactly $a_i$ vertices of degree $\pi(a_i)$. It has been shown that for positive integers with $a_1 < a_2 < ... < a_n$, if $\pi(a_n) = a_n$, then the permutation $\pi$ is graphical if and only if the sum $\sum_{i=1}^{n} a_i \pi(a_i)$ is even and $a_n \leq \sum_{i=1}^{n-1} a_i \pi(a_i)$. This known result has been proved using a criterion of Fulkerson, Hoffman, and McAndrew which requires the verification of $\frac{1}{2}n(n+1)$ inequalities. In this talk, we use a criterion of Tripathi and Vijay to give a shorter proof of this result, requiring only the verification of $n$ inequalities. We also use this criterion to provide a similar result for permutations $\pi$ such that $\pi(a_{n-1}) = a_n$. We prove that such a permutation is graphical if and only if the sum $\sum_{i=1}^{n} a_i \pi(a_i)$ is even and $a_n a_{n-1} \leq a_{n-1}(a_{n-1} - 1) + \sum_{i \neq n-1} a_i \pi(a_i)$. We also consider permutations such that $\pi(a_n) = a_{n-1}$, and then more generally, permutations such that $\pi(a_n) = a_{n-j}$ for some $j < n$. A Study of Graphical Permutations

Jessica Thune*

Department of Mathematical Sciences, University of Nevada Las Vegas
Las Vegas, NV 89154
thunej@unlv.nevada.edu

A permutation $\pi$ on a set of positive integers $\{a_1, a_2, ..., a_n\}$ is said to be graphical if there exists a graph containing exactly $a_i$ vertices of degree $\pi(a_i)$. It has been shown that for positive integers with $a_1 < a_2 < ... < a_n$, if $\pi(a_n) = a_n$, then the permutation $\pi$ is graphical if and only if the sum $\sum_{i=1}^{n} a_i \pi(a_i)$ is even and $a_n \leq \sum_{i=1}^{n-1} a_i \pi(a_i)$. This known result has been proved using a criterion of Fulkerson, Hoffman, and McAndrew which requires the verification of $\frac{1}{2}n(n+1)$ inequalities. In this talk, we use a criterion of Tripathi and Vijay to give a shorter proof of this result, requiring only the verification of $n$ inequalities. We also use this criterion to provide a similar result for permutations $\pi$ such that $\pi(a_{n-1}) = a_n$. We prove that such a permutation is graphical if and only if the sum $\sum_{i=1}^{n} a_i \pi(a_i)$ is even and $a_n a_{n-1} \leq a_{n-1}(a_{n-1} - 1) + \sum_{i \neq n-1} a_i \pi(a_i)$. We also consider permutations such that $\pi(a_n) = a_{n-1}$, and then more generally, permutations such that $\pi(a_n) = a_{n-j}$ for some $j < n$.